Homework 11

Due Friday, May 5 at 10pm. Please upload a legible pdf to Gradescope.

You may work together, but the solutions must be written up in your own words.

§# refer to Brown and Churchill, Ninth Edition

Hand in:

§94 #5, #6 (a) only

Additional #1 Prove that there does not exists a conformal map from $\mathbb{C}$ to the unit disc $\{|z| < 1\}$. Hint: Liouville.

Additional #2
Let f, g be fractional linear transformations

$$f(z) = \frac{az+b}{cz+d} \quad g(z) = \frac{Az+B}{Cz+D}.$$

Relate the fractional linear transformation $f \circ g$ to the matrix product $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} A & B \\ C & D \end{pmatrix}$.

Additional #2
Let D be the interior of the closed contour formed by C_1 = the left half of the unit circle $\{e^{i\theta} | \pi/2 \leq \theta \leq 3\pi/2\}$ and C_2 = the line segment between $-i$ and i.

a) Find a Fractional Linear Transofrmation $f(z)$ such that $f(i) = 0$ and $\lim_{z \to -i} f(z) = \infty$. There are many correct answers.

b) $f(D)$ will be the region between two rays $re^{i\theta_0}$ and $re^{i\theta_1}$ through the origin. Identify θ_0 and θ_1.

c) By composing an anlaytic branch of log and f, and manipulating the imaginary part of the composition, describe a harmonic function on D with limit 1 on C_1 and 2 on C_2. No need for an explicit formula.

Recommended, but don't hand in:
Let D be a simply connected domain which does not contain 0, and let C be a closed contour in D. Prove that the winding number of C is 0. Hint: Show that there is a continuous branch of log on D.