

1) Ross 4.11

2) Ross 4.15

3) Ross 8.4

4) Let  $|a| < 1$ ,  $p \in \mathbb{N}$ . Prove  
 $a^n n^p \rightarrow 0$ .

5) Give an example of a sequence  
 $(s_n)$  which does not converge  
such that  $(t_k) = (\frac{1}{k}) = (1, \frac{1}{2}, \frac{1}{3}, \dots)$   
is a subsequence of  $(s_n)$

6) Ross 11.10

7) Ross 11.11

8) Find lim sup and lim inf of  $\cos(\frac{n\pi}{3})$

9) Give an example of a sequence  $(S_n)$  such that  $\limsup S_n > 0$  with a subsequence  $(t_k)$  such that  $t_k < 0$  for all  $k \in \mathbb{N}$ .

10) Prove, directly from the definition, that  $(a, b)$  is open and  $[a, b]$  is closed for all  $a, b \in \mathbb{R}$ .

11) Let  $S$  be a metric space with distance function  $d$ . Let  $r \in S$ ,  $\varepsilon \in \mathbb{R}$ ,  $\varepsilon > 0$ .

a) Prove that

$$B_\varepsilon(r) = \{s \in S \mid d(s, r) < \varepsilon\}$$

is an open set.

b) Prove that

$$\{s \in S \mid d(s, r) \leq \varepsilon\}$$

is a closed set.

12) Ross 14.12

13) Prove that  $\sum \frac{1}{\sqrt{n}}$  does not converge

14) Prove that  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = \begin{cases} \frac{x}{|x|} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

is not continuous.

a) using the definition (sequences) b) using  $\varepsilon$ - $\delta$  property

15) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function and  $(S_n)$  a sequence such that  $f(S_n) = S_{n+1}$  for all  $n \in \mathbb{N}$  and  $S_n \rightarrow S$ .  
Prove that  $f(S) = S$ .

16) Let  $f: S \rightarrow \mathbb{R}$  be a function and  $E$  an open set such that  $E \subseteq S$ .  
Define  $g: E \rightarrow \mathbb{R}$  by  $g(x) = f(x)$  for all  $x \in E$  ( $g$  is the restriction of  $f$ ).  
Assume  $g$  is continuous at  $x_0 \in E$ .  
Prove  $f$  is continuous at  $x_0$ .

17) Prove that  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = |x|$  is continuous  
a) using the definition (sequences)  
b) using  $\epsilon - \delta$  property.

18) Ross 17, 8

19) Ross 17, 9

20) Prove  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x^2$  is  
not uniformly continuous.

21) Ross 18.10

22) Ross 19.2

23) Ross 19.3

24) Prove that  $f: (0,1) \rightarrow \mathbb{R}$ ,  $f(x) = \frac{1}{x^2}$   
is not uniformly continuous.