Lecture 25, 4/21/21
Material corresponds to Ross §26, Rudin §7.

Integrating and Differentiating Sequences of Functions

Theorem let $f_n, f : [a, b] \to \mathbb{R}$ be functions for all $n \in \mathbb{N}$ such that $f_n \to f$ uniformly. If f_n is integrable for all $n \in \mathbb{N}$ then f is integrable and

$$\lim_{n\to\infty} \int_a^b f_n = \int_a^b f.$$

Theorem Let $f_n : [a, b] \to \mathbb{R}$ be continuous and differentiable on (a, b) for all $n \in \mathbb{N}$. Assume there exists $x_0 \in [a, b]$ such that $(f_n(x_0))$ converges and $g : (a, b) \to \mathbb{R}$ such that $f'_n \to g$ uniformly. Then there exists $f : [a, b] \to \mathbb{R}$ such that $f_n \to f$ uniformly, f is differentiable on (a, b), and $f'(x) = g(x)$ for all $x \in (a, b)$.

$$(\text{i.e.}(\lim f_n)' = \lim f'_n)$$