Lecture 14, 3/3/22
Material corresponds to Ross §$18-19$.

Properties of Continuous Functions

Extreme Value Theorem Let S be sequentially compact and let $f : S \to \mathbb{R}$ be continuous. Then there exists $x_0, x_1 \in S$ such that $f(x_0) \leq f(x) \leq f(x_1)$ for all $x \in S$.

Intermediate Value Theorem Let $I \subseteq \mathbb{R}$ be an interval and let $f : I \to \mathbb{R}$ be continuous. If $a, b \in I$, $a < b$, and y lies between $f(a)$ and $f(b)$ (i.e. either $f(a) < y < f(b)$ or $f(b) < y < f(a)$) then there exists $x \in (a, b)$ such that $f(x) = y$.

Uniform Continuity

Definition A function $f : S \to \mathbb{R}$ is **uniformly continuous** if for every $\epsilon > 0$ there exists $\delta > 0$ such that $x, y \in S$ and $|x - y| < \delta$ implies $|f(x) - f(y)| < \epsilon$.

Theorem If S is sequentially compact and $f : S \to \mathbb{R}$ is continuous then f is uniformly continuous.