Lecture 15, 3/29/22
Material corresponds to Ross §28.

Derivatives

Definition Let $I \subset \mathbb{R}$ be an open interval, $a \in I$, and $f : I \to \mathbb{R}$.

1. the **difference quotient** of f at a is
$$\frac{f(x) - f(a)}{x - a} : I \backslash \{a\} \to \mathbb{R}.$$

2. f is **differentiable at** a if
$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$
exists. If the limit exists, it is called $f'(a)$, "the derivative of f at a".

3. f is **differentiable** if it is differentiable at all $a \in I$. In that case $f' : I \to \mathbb{R}$ is a function called "the derivative of f."

Theorem Let $I \subset \mathbb{R}$ be an open interval, $f : I \to \mathbb{R}$. If f is differentiable at $a \in I$ then f is continuous at $a \in I$.

Derivative Theorems Let $I \subset \mathbb{R}$ be an open interval, and let $f, g : I \to \mathbb{R}$ be differentiable at $a \in I$. Let $c \in \mathbb{R}$ and define $h : \mathbb{R} \backslash \{0\} \to \mathbb{R}$ by $h(x) = \frac{1}{x}$. Then

1. cf is differentiable at a and $(cf)'(a) = c(f'(a))$
2. $f + g$ is differentiable at a and $(f + g)'(a) = f'(a) + g'(a)$
3. fg is differentiable at a and $(fg)'(a) = f'(a)g(a) + f(a)g'(a)$
4. h is differentiable and $h'(a) = -\frac{1}{a^2}$ for all $a \neq 0$.

Theorem (Chain Rule) Let $I_1 \subset \mathbb{R}$ be an open interval, $a \in I_1$, $f : I_1 \to \mathbb{R}$ differentiable at a. Let $I_2 \subset \mathbb{R}$ be an open interval such that $f(I_1) \subseteq I_2$ and let $g : I_2 \to \mathbb{R}$ be differentiable at $f(a)$. Then $g \circ f : I_1 \to \mathbb{R}$ is differentiable at a and $(g \circ f)'(a) = g'(f(a))f'(a)$.