Lecture 17, 3/17/22
Material corresponds to Ross §13, 22.

Let S be a metric space with distance function d. Let S^* be a metric space with distance function d^*.

Connectedness

Theorem If $E \subset S$ is connected and $f : S \to S^*$ is continuous then $f(E)$ is connected.

Corollary If $E \subset S$ is connected and $f : E \to \mathbb{R}$ is continuous, then $f(I)$ is an interval. In particular, if $a, b \in E$ and $f(a) < y < f(b)$ then there exists $x \in E$ such that $f(x) = y$.

Compactness

Definition

1. An **open cover** of $E \subseteq S$ is a collection C of open sets $U \subset S$ such that ever element of E is contained in some U, that is, $E \subseteq \bigcup_{U \in C} U$.
2. A **finite subcover** $C' \subset C$ is a finite subset $C' = \{U_{\alpha_1}, ..., U_{\alpha_k}\} \subset C$ such that which is still an open cover of E, that is, $E \subseteq U_{\alpha_1} \cup ... \cup U_{\alpha_k}$.

Definition A set $E \subseteq S$ is **compact** if every open cover of E contains a finite subcover.

Theroem A set $E \subseteq S$ is compact if and only if it is sequentially compact.