Lecture 6, 2/3/22
Material corresponds to Ross §9 − 10.

Limit Theorems

Theorem If $s_n \to s$ and $t_n \to t$ then $s = t$

Limit Theorems, Ross 9.2-9.6 Let $s_n \to s$ and $t_n \to t$ in $\mathbb{R}$. Then:

1. For all $k \in \mathbb{R}$, $ks_n \to ks$.
2. $s_n + t_n \to s + t$.
3. $s_n t_n \to st$.
4. If $s_n \neq 0$ for all $n \in \mathbb{N}$ and $s \neq 0$ then $1/s_n \to 1/s$ and $t_n/s_n \to t/s$.

Definition A sequence (s_n) is **bounded** if its set of values $\{s_n\}$ is bounded.

Theorem If $s_n \to s$ then $\{s_n\}$ is bounded.

Theorem Let $a, b \in \mathbb{R}$. If $a \leq s_n \leq b$ for all $n \in \mathbb{N}$ and $s_n \to s$ then $a \leq s \leq b$.

Theorem (Squeeze Theorem) If $a_n \leq s_n \leq b_n$ for all $n \in \mathbb{N}$, $a_n \to s$, and $b_n \to s$ then $s_n \to s$.

Important Limits, Ross 9.7

1. If $p > 0$, $\lim_{n\to\infty} \frac{1}{n^p} = 0$
2. If $|a| < 1$, $\lim_{n\to\infty} a^n = 0$
3. $n^{1/n} \to 1$
4. If $a > 0$, $\lim_{n\to\infty} a^{1/n} = 1$

Monotone Sequences

Defenition Let (s_n) be a sequence in $\mathbb{R}$.

1. (s_n) is **increasing** if $s_n \leq s_{n+1}$ for all $n \in \mathbb{N}$.
2. (s_n) is **decreasing** if $s_n \geq s_{n+1}$ for all $n \in \mathbb{N}$.
3. (s_n) is **monotone** if it is increasing or decreasing.

Theorem A bounded monotone sequence converges. If (s_n) is bounded and increasing (or decreasing) and $s_n \to s$ then $s_n \leq s$ (or $s_n \geq s$) for all $n \in \mathbb{N}$.

Tails

Definition Let (s_n) be a sequence. A **tail** of (s_n) is a set of the form

$$T_n = \{s_n | n > N\}$$

for some $N \in \mathbb{R}$.

Fact $s_n \to s$ if and only if for each $\epsilon > 0$ there exists $N \in \mathbb{R}$ such that $T_N \subset (s - \epsilon, s + \epsilon)$.