Lecture 9, 2/15/22
Material corresponds loosely to Ross §13.

Open and Closed sets

Definition 1

- A set $E \subset \mathbb{R}$ is closed if for every sequence (s_n) in E, $s_n \to s$ implies $s \in E$.
- A set E is open if its complement $\mathbb{R} \backslash E = \{s \in \mathbb{R} | s \notin E\}$ is closed.

Definition 2

- A set $E \subset \mathbb{R}$ is open if for all $s \in E$ there exists $\epsilon > 0$ such that $(s - \epsilon, s + \epsilon) \subset E$.
- A set E is closed if its complement $\mathbb{R} \backslash E = \{s \in \mathbb{R} | s \notin E\}$ is open.

Theorem The two definitions are equivalent: A set is open (definition 2) if and only if its complement is closed (definition 1).

Compactness

Definition A set $E \subset \mathbb{R}$ is **sequentially compact** if every sequence (s_n) in E has a subsequence which converges to an element of E.