Lecture 8, 2/10/22
Material corresponds to Ross §11.

Subsequences

Defenition Let (s_n) be a sequence. A sequence (t_k) is a **subsequence** of (s_n) if for each $k \in \mathbb{N}$ there exists $n_k \in \mathbb{N}$ such that $t_k = s_{n_k}$ and $n_1 < n_2 < ... < n_k < n_{k-1} < ...$

Lemma In the above situation, $n_k \geq k$.

Theorem If $s_n \to s$ any subsequence of (s_n) converges to s.

Lemma Let s_n be a bounded sequence. Let $r_0 = \limsup s_n$. Let $\epsilon > 0$.

1. The set $\{n \in \mathbb{N} | \ r_0 - \epsilon < s_n\}$ is unbounded.
2. The set $\{n \in \mathbb{N} | \ r_0 + \epsilon \leq s_n\}$ is finite.
3. The set $\{n \in \mathbb{N} | \ |s_n - r_0| < \epsilon\}$ is unbounded.

Theorem (Bolzano-Weierstrass) Every bounded sequence in $\mathbb{R}$ has a convergent subsequence.