Lecture 3, 1/25/22
Material corresponds to Ross §4, and the end of §3.

Fact: Let $S \subseteq \mathbb{R}$ have a supremum and let $a \in \mathbb{R}$ (a need not be in S). If $a < \sup S$ then there exists $r \in S$ such that $a < r \leq \sup S$.

Completeness Axiom If $S \subset \mathbb{R}$ is nonempty and bounded above then S has a supremum.

Corollary If $S \subset \mathbb{R}$ is nonempty and bounded below then S has an infimum.

Archimedean Principle Let $a \in \mathbb{R}$. Then there exists $n \in \mathbb{N}$ such that $a < n$.

Denseness of $\mathbb{Q}$ Let $a, b \in \mathbb{R}$, $a < b$. Then there exists $r \in \mathbb{Q}$ such that $a < r < b$.

Denseness of irrational numbers Let $a, b \in \mathbb{R}$, $a < b$. Then there exists $r \notin \mathbb{Q}$ such that $a < r < b$.

Absolute Value

Definition For each $a \in \mathbb{R}$, $|a| = a$ if $a \geq 0$ and $|a| = -a$ if $a < 0$.

Theorem

1. $|a| \geq 0$
2. $|ab| = |a||b|$
3. (Triangle inequality) $|a + b| \leq |a| + |b|$

Useful facts

- $-|a| \leq a \leq |a|$
- $|a + b| \geq |a| - |b|$
- If $c > 0$

$$|a - b| < c \quad \text{if and only if} \quad b - c < a < b + c \quad \text{if and only if} \quad a \in (b - c, b + c)$$