Lecture 1, 1/18/22
Material corresponds to Ross §1 and §3. I also reccomend §2 and the Appendix on Set Notation.

Numbers

<u>Sets</u>	<u>Axioms</u>
$\mathbb{N} = \{1, 2, 3, ...\}$	Peano Axioms
$\cap$	
$\mathbb{Z} = \{..., -2, -1, 0, 1, 2, ...\}$	
$\cap$	
$\mathbb{Q} = \{p/q \mid p, q \in \mathbb{Z}, q \neq 0\}$	Ordered Field Axioms
$\cap$	
$\mathbb{R} = ...$	Ordered Field Axioms, Completeness Axiom

Axioms for $\mathbb{N}$ (Peano Axioms) (Ross p.1)

1. $1 \in \mathbb{N}$
2. If $n \in \mathbb{N}$ then $n + 1 \in \mathbb{N}$
3. If $n \in \mathbb{N}$ then $n + 1 \neq 1$
4. If $n + 1 = m + 1$ then $n = m$
5. (Induction Axiom) Let $S \subseteq \mathbb{N}$ be a set with the following properties:
    - $1 \in S$
    - If $n \in S$ then $n + 1 \in S$

    Then $S = \mathbb{N}$.

Ordered Field Axioms (Ross p.13)

The following hold with $\mathbb{Q}$ replaced by $\mathbb{R}$ as well:

Let $a, b \in \mathbb{Q}$. Then $a + b \in \mathbb{Q}$ and $ab \in \mathbb{Q}$.

- Addition is Commutative, Associative, has 0, and has additive inverses:
  $a + b = b + a$, $(a + b) + c = a + (b + c)$, $a + 0 = a$ and $a + (-a) = 0$.

- Multiplication is commutative, associative, distributive, has 1, and has multiplicative inverses (except 0):
  $ab = ba$, $(ab)c = a(bc)$, $a(b + c) = ab + ac$, $a \cdot 1 = a$ and if $a \neq 0$, and $a(a)^{-1} = 1$.

For all $a, b, c \in \mathbb{Q}$

- Either $a \leq b$, $b \leq a$, or both.
- If $a \leq b$ and $b \leq a$ then $a = b$.
- If $a \leq b$ and $b \leq c$ then $a \leq c$.
- If $a \leq b$ then $a + c \leq b + c$
- If $a \leq b$ and $0 \leq c$ then $ac \leq bc$.

Facts following from Ordered Field Axioms (Ross p.15)

Theorem 1 Let $a, b, c \in \mathbb{R}$

1. If $a + c = b + c$ then $a = b$
2. $a0 = 0$
3. $(-a)b = -(ab)$
4. $(-a)(-b) = ab$
5. If $ac = bc$ and $c \neq 0$ then $a = b$
6. If $ab = 0$ then $a = 0$ or $b = 0$.

Theorem 2 Let $a, b, c \in \mathbb{R}$.

1. If $a \leq b$ then $-b \leq -a$
2. If $a \leq b$ and $c \leq 0$ then $bc \leq ac$
3. If $0 \leq a$ and $0 \leq b$ then $0 \leq ab$
4. $0 \leq a^2$
5. $0 < 1$
6. If $0 < a$ then $0 \leq a^{-1}$
7. If $0 < a < b$ then $0 < b^{-1} < a^{-1}$.