

Homework 8

Due Tuesday, April 12 at 10am. Please upload a legible copy to Gradescope.

You may work together, but the solutions must be written up in your own words. Show all work and justify all answers.

1. Ross 29.2
2. Ross 29.5
3. Ross 29.13
4. Ross 29.17
5. Ross 30.1
6. Ross 30.2. You may assume that \log is continuous.
7.
 - a) Let (s_n) be a sequence such that $s_n \neq 0$ for all $n \in \mathbb{N}$. Prove that $|s_n| \rightarrow \infty$ if and only if $\frac{1}{|s_n|} \rightarrow 0$.
 - b) Let $f : E \rightarrow \mathbb{R}$ be a function such that $f(x) \neq 0$ for all $x \in E$. Let a be the limit of a sequence in E . Prove that $\lim_{x \rightarrow a} |f(x)| = \infty$ if and only if that $\lim_{x \rightarrow a} \frac{1}{f(x)} = 0$.
 - c) Let $a > 0$, $f : (a, \infty) \rightarrow \mathbb{R}$ a function, and $g : (0, \frac{1}{a}) \rightarrow \mathbb{R}$ the function defined by $g(x) = f(\frac{1}{x})$. Prove that $\lim_{x \rightarrow \infty} f(x) = L$ if and only if $\lim_{x \rightarrow 0} g(x) = L$.
8.
 - a) Let $E \subseteq \mathbb{R}$ and let a be the limit of a sequence in E . Let $f, g : E \rightarrow \mathbb{R}$ be functions such that $f(x) \leq g(x)$ for all $x \in E$, $\lim_{x \rightarrow a} f(x) = F$ and $\lim_{x \rightarrow a} g(x) = G$. Prove that $F \leq G$.
 - b) The fact in part a becomes false if we replace \leq with $<$. Find f, g demonstrating this.