Homework 8

Due Tuesday, April 5 at 10am. Please upload a legible copy to Gradescope.

You may work together, but the solutions must be written up in your own words. Show all work and justify all answers.

1. Prove that $\mathbb{R}$ is not compact. Note: use only the definition of compactness; do not use sequential compactness.

2. Let S be a metric space with distance function d. Let E be a compact subset of S. Let F be a closed subset of S such that $F \subseteq E$. Prove that F is compact. Note: use only the definition of compactness; do not use sequential compactness.

3. Ross 28.2

4. Ross 28.3

5. Ross 28.4

6. Ross 28.10. You may use the derivatives of $\cos(x)$ and e^x without proof.

7. a) Use the product rule and induction to show that $(x^n)' = nx^{n-1}$ for all $n \in \mathbb{N}$.

   b) Use the fact that $\left(\frac{1}{x}\right)' = \left(-\frac{1}{x^2}\right)$ and the chain and product rules to prove the quotient rule: If $I \subseteq \mathbb{R}$ is an open interval, $f, g : I \to \mathbb{R}$ are differentiable at $a \in I$, and $g(x) \neq 0$ for $x \in I$, then
$$\left(\frac{f}{g}\right)'(a) = \frac{f'(a)g(a) - f(a)g'(a)}{(g(a))^2}.$$