

# Homework 10

Due Tuesday, April 19 at 10am. Please upload a legible copy to Gradescope.

You may work together, but the solutions must be written up in your own words. Show all work and justify all answers.

1. Ross, 32.6
2. Use the previous problem to show that  $f : [a, b] \rightarrow \mathbb{R}$ ,  $f(x) = x$  is integrable, and find  $\int_a^b f$ .
3. Define  $f : [0, 1] \rightarrow \mathbb{R}$  by  $f(x) = x$  if  $x \in \mathbb{Q}$  and  $f(x) = 0$  if  $x \notin \mathbb{Q}$ . Compute  $U(f)$  and  $L(f)$ . Is  $f$  integrable? Hint: Compare  $U(f)$  to the same quantity for a simpler function.
4. Ross 32.8
5. Let  $a, b \in \mathbb{R}$ ,  $a < b$ . Let  $f : [a, b] \rightarrow \mathbb{R}$  be a function such that  $f(x) = 0$  for all  $x \in [a, b] \setminus S$ , where  $S = \{s_1, \dots, s_k\}$  is a finite subset of  $[a, b]$ . Prove that  $f$  is integrable and find  $\int_a^b f$ .
6. Ross 33.7 and 33.8 (a).
7. Define  $\text{sgn} : \mathbb{R} \rightarrow \mathbb{R}$  by  $\text{sgn}(x) = 1$  if  $x > 0$ ,  $\text{sgn}(x) = -1$  if  $x < 0$ , and  $\text{sgn}(0) = 0$ . Define  $f : [-1, 1] \rightarrow \mathbb{R}$  by  $f(x) = x \text{sgn}(\sin(\frac{1}{x}))$  for  $x \neq 0$  and  $f(0) = 0$ .
  - a) At what points  $x \in [-1, 1]$  is  $f$  continuous?
  - b) Prove that  $f$  is integrable. Hint:  $f$  is not piecewise continuous on all of  $[-1, 1]$ .