14)

a) Let $s_n = \frac{1}{n}$. Then $s_n \in \mathbb{R}$, $s_n \to 0$

but $f(s_n) = \frac{1/n}{1/n} = 1$ does not converge to

$f(0) = 0$.

b) Choose $\varepsilon = 1/2$. Let $\delta > 0$. Choose $x = \frac{\delta}{2}$.

Then $|x - 0| = \frac{\delta}{2} < \delta$ but

$|f(x) - f(0)| = \left| \frac{\delta/2}{\delta/2} - 0 \right| = 1 > \frac{1}{2} = \varepsilon$.

17)

a) Let $x_0 \in \mathbb{R}$, $s_n \to x_0$.

Let $\varepsilon > 0$. There exists $N \in \mathbb{R}$ such that

$n > N$ implies $|s_n - x_0| < \varepsilon$.

Thus for $n > N$,

$$\big|\,|s_n| - |x_0|\,\big| = \begin{cases} |s_n| - |x_0| \\ \text{or } |x_0| - |s_n| \end{cases} \leq |s_n - x_0| < \varepsilon.$$

($\triangle$ inequality; see also HW 1)

So $f(s_n) = |s_n| \to |x_0| = f(x_0)$.

b) Let $x_0 \in \mathbb{R}$, $\varepsilon > 0$. Choose $\delta = \varepsilon$.

If $|x - x_0| < \delta$, then

$$|f(x) - f(x_0)| = \big|\,|x| - |x_0|\,\big| \leq |x - x_0| < \delta = \varepsilon.$$

18) Ross 17, 8

a) If $f(x) \geq g(x)$ then

$$\frac{1}{2}(f(x)+g(x)) - \frac{1}{2}|f(x)-g(x)| = \frac{1}{2}(f(x)+g(x)-f(x)+g(x)) = g(x)$$
$$= \min\{f(x), g(x)\}.$$

If $f(x) < g(x)$

$$\frac{1}{2}(f(x)+g(x)) - \frac{1}{2}|f(x)-g(x)| = \frac{1}{2}(f(x)+g(x)+f(x)-g(x))$$
$$= f(x) = \min\{f(x), g(x)\}$$

c) Since f, g are continuous, $f-g$, $\frac{1}{2}(f+g)$ are continuous. Since $|\cdot|$ is continuous (#17) $|f-g|$ is continuous (composition), so

$$\min(f,g) = \frac{1}{2}(f+g) - \frac{1}{2}|f-g|$$

is continuous.

19) Ross 17, 9

c) Let $\varepsilon > 0$. Let $\delta = \varepsilon$

If $x \in \mathbb{R}$, $x \neq 0$, $|x - 0| < \delta$, then

$|f(x) - f(0)| = |x \sin(\frac{1}{x}) - 0| = |x| \, |\sin(\frac{1}{x})| \leq |x| < \delta = \varepsilon$.