1) Ross 4.11

2) Ross 4.15

3) Ross 8.4

4) Let $|a|<1$, $p \in \mathbb{N}$. Prove
$$a^n n^p \to 0.$$

5) Give an example of a sequence (s_n) which does not converge such that $(t_k) = (\frac{1}{k}) = (1, \frac{1}{2}, \frac{1}{3}, \dots)$ is a subsequence of (s_n)

6) Ross 11.10

7) Ross 11.11

8) Find lim sup and lim inf of $\cos(\frac{n\pi}{3})$

9) Give an example of a sequence (s_n) such that $\limsup s_n > 0$ with a subsequence (t_k) such that $t_k < 0$ for all $k \in \mathbb{N}$.

10) Prove, directly from the definition, that (a, b) is open and $[a, b]$ is closed for all $a < b \in \mathbb{R}$.

12) Ross 14.12

13) Prove that $\sum_{1}^{\infty} \frac{1}{\sqrt{n}}$ does not converge

14) Prove that $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} \frac{x}{|x|} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

is not continuous.

a) using the definition (sequences) b) using ε-δ property

15) Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous function, and (S_n) a a sequence such that $f(S_n) = S_{n+1}$ for all $n \in \mathbb{N}$ and $S_n \to S$. Prove that $f(S) = S$.

16) Let $f: S \to \mathbb{R}$ be a function and E an open set such that $E \subseteq S$. Define $g: E \to \mathbb{R}$ by $g(x) = f(x)$ for all $x \in E$ (g is the restriction of f). Assume g is continuous at $x_0 \in E$. Prove f is continuous at x_0.

17) Prove that $f: \mathbb{R} \to \mathbb{R}$, $f(x) = |x|$ is continuous

a) using the definition (sequences)

b) using $\varepsilon - \delta$ property.

18) Ross 17, 8

19) Ross 17, 9