Lecture 6, 9/7/22
Material corresponds to Ross §$7-8$.

Sequences

Theorem (Triangle inequality) If $a, b, c \in \mathbb{R}$ then

$$|a-b| \leq |a-c| + |c-d|.$$

Theorem (Limits are Unique) If $s_n \to s$ and $s_n \to t$ then $s = t$.

Example

$$\frac{2n^2-7n}{n^2-n+8} \to 2$$

Let $\epsilon > 0$. Choose $N = \frac{21}{\epsilon} + 1$. Let $n > N$. Then

$$\left|\frac{2n^2-7n}{n^2-n+8} - 2\right| = \left|\frac{5n+16}{n^2-n+8}\right| \leq \frac{5n+16}{n^2-n} \leq \frac{21n}{n^2-n} = \frac{21}{n-1}$$

because $n > 1$, , $|n^2-n+8| = n^2-n+8 \geq n^2-n$ and $5n+16 \leq 5n+16n$. Furthermore, Finally, since $n > N = \frac{21}{\epsilon} + 1$,

$$\frac{21}{n-1} < \epsilon.$$

Stringing the inequalities together we have

$$\left|\frac{2n^2-7n}{n^2-n+8} - 2\right| < \epsilon.$$