Lecture 5, 9/2/22
Material corresponds to Ross §7.

Sequences

A sequence is an infinite list of numbers $s_n \in \mathbb{R}$:

$$(s_m, s_{m+1}, s_{m+2}, ..., s_n, ...) = (s_n)_{n=m}^{\infty}.$$

s_n is an "element" of the sequence, and n is the "index" of s_n. The first index m can be any integer. If it is not specified, the default is 1:

$$(s_n) = (s_n)_{n=1}^{\infty} = (s_1, s_2, s_3, ..., s_n, ...).$$

The **set of values** of s_n is the set $\{s_1, s_2, s_3, ..., s_n, ...\}$.

Definition A sequence (s_n) **converges to** $s \in \mathbb{R}$ if for every $\epsilon > 0$ there exists an $N \in \mathbb{R}$ such that $n > N$ implies $|s_n - s| < \epsilon$.

We write $s_n \to s$, $\lim_{n\to\infty} s_n = s$, or $\lim s_n = s$. s is called the limit of (s_n).

Fact A sequence (s_n) *does not converge* to $s \in \mathbb{R}$ if there exists $\epsilon > 0$ such that for all $N \in \mathbb{R}$ there exists $n > N$ such that $|s_n - s| \geq \epsilon$.