Homework 2

Due Monday, September 12 at 10am. Please upload a legible copy to Gradescope.

You may work together, but the solutions must be written up in your own words. Show all work and justify all answers.

1. What is the definition of the supremum of a set? What is the definition of the infimum of a set? Give an example of

   a) A set $S \subseteq \mathbb{R}$ such that $\sup S \in S$

   b) A set $S \subseteq \mathbb{R}$ such that $\sup S \in \mathbb{R} \backslash S$

   c) A set $S \subseteq \mathbb{R}$ which does not have a supremum.

2. Ross, 4.11.

3. Ross, 4.15.

4. Prove that $S = \{r \in \mathbb{R} | r^2 < 2\}$ has a supremum. If $s = \sup S$, prove that $s^2 = 2$. Do not use the number $\sqrt{2}$ in your proof.

5. a) Find $N_1 \in \mathbb{R}$ such that if $n > N_1$, then $n + 6 \leq 7n$

   b) Find $N_2 \in \mathbb{R}$ such that if $n > N_2$, then $n^2 - 6 \geq \frac{n^2}{2}$

   c) Find $N_3 \in \mathbb{R}$ such that if $n > N_3$, $\left|\frac{n+6}{n^2-6}\right| < \frac{14}{n}$.

   d) Let $\epsilon > 0$. Find $N_4 \in \mathbb{R}$ such that if $n > N_4$, $14/n < \epsilon$.

   e) Use parta a-d to prove, directly from the definition of convergence of a sequence, that

$$\frac{n+6}{n^2-6} \to 0.$$

6. a) Find $N_1 \in \mathbb{R}$ such that if $n > N_1$, then $n + 6 \leq 7n$

   b) Find $N_2 \in \mathbb{R}$ such that if $n > N_2$, then $n^2 - 6 \geq n^2 - 6n > 0$

   c) Find $N_3 \in \mathbb{R}$ such that if $n > N_3$, $\left|\frac{n+6}{n^2-6}\right| < \frac{7}{n-6}$.

   d) Let $\epsilon > 0$. Find $N_4 \in \mathbb{R}$ such that if $n > N_4$, $\frac{7}{n-6} < \epsilon$.

   e) Use parta a-d to prove, directly from the definition of convergence of a sequence, that

$$\frac{n+6}{n^2-6} \to 0.$$

7. Ross 8.1. Prove that the sequence converges to that limit directly from the definition; do not use any limit theorems.

8. Ross 8.2. Prove that the sequence converges to that limit directly from the definition; do not use any limit theorems.