

Homework 11

Due Monday, November 28 at 10am. Please upload a legible copy to Gradescope.

You may work together, but the solutions must be written up in your own words. Show all work and justify all answers.

1. Ross 32.8
2. Let $a, b \in \mathbb{R}$, $a < b$. Let $f : [a, b] \rightarrow \mathbb{R}$ be a function such that $f(x) = 0$ for all $x \in [a, b] \setminus S$, where $S = \{s_1, \dots, s_k\}$ is a finite subset of $[a, b]$. Prove that f is integrable and find $\int_a^b f$.
3. Ross 33.7 and 33.8 (a).
4. Let $f : [a, b] \rightarrow \mathbb{R}$ be a continuous function such that $f(x) \geq 0$ for all $x \in [a, b]$ and $\int_a^b f = 0$. Prove that $f(x) = 0$ for all $x \in \mathbb{R}$.
5. Ross 34.2
6. Define $f : \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = 0$ for $x < 0$, $f(x) = x$ for $x \in [0, 1]$ and $f(x) = 4$ for $x > 1$. Define $F : \mathbb{R} \rightarrow \mathbb{R}$ by $F(x) = \int_0^x f$.
 - a) Determine $F(x)$ for each $x \in \mathbb{R}$.
 - b) At which $x \in \mathbb{R}$ is F continuous?
 - c) At which $x \in \mathbb{R}$ is F differentiable? For those x , what is $F'(x)$?
7. Ross 34.5
8. Ross 34.7. Indicate precisely how you use change of variables, and check that all conditions of the theorem are met.