Lecture 5, 9/9/21
Material corresponds to Ross §10.

Bounded and Monotone Sequences

Defenition Let (s_n) be a sequence in $\mathbb{R}$.

1. (s_n) is **bounded** if $\{s_n\}$ is bounded.
2. (s_n) is **nondecreasing** if $s_n \leq s_{n+1}$ for all $n \in \mathbb{N}$.
3. (s_n) is **nonincreasing** if $s_n \geq s_{n+1}$ for all $n \in \mathbb{N}$.
4. (s_n) is **monotone** if it is nondecreasing or nonincreasing.

> **Theorem** A bounded monotone sequence converges.

$\limsup$ and $\liminf$

Definition For a bounded sequence (s_n) in $\mathbb{R}$:

$$\limsup s_n = \lim_{N \to \infty} \sup\{s_n | n > N\}$$

$$\liminf s_n = \lim_{N \to \infty} \inf\{s_n | n > N\}$$

> **Theorem** Let (s_n) be a bounded sequence in $\mathbb{R}$. Then (s_n) converges if and only if $\liminf s_n = \limsup s_n$. If s_n converges then $\lim s_n = \liminf s_n = \limsup s_n$.

Cauchy Sequences

Definition A sequence (s_n) is Cauchy if for all $\epsilon > 0$ there exists $N \in \mathbb{R}$ such that if $n, m > N$ then $|s_n - s_m| < \epsilon$.

Lemma A Cauchy sequence is bounded.

> **Theorem** A sequence in $\mathbb{R}$ converges if and only if it is Cauchy.