Lecture 3, 9/2/21
Material corresponds to Ross §7 and §13.

Metric Space

A metric space is a set S together with a distane function d such that for $a, b \in S$, $d(a, b) \in \mathbb{R}$ and

1. $d(a, b) \geq 0$ and $d(a, b) = 0$ if and only if $a = b$.
2. $d(a, b) = d(b, a)$.
3. $d(a, b) \leq d(a, c) + d(c, b)$ for all $c \in S$.

$\mathbb{R}$ is a metric space with $d(a, b) = |a - b|$ for all $a, b \in \mathbb{R}$.

Sequences

A sequence is an infinite list of numbers $s_n \in \mathbb{R}$:

$$(s_m, s_{m+1}, s_{m+2}, ..., s_n, ...) = (s_n)_{n=m}^{\infty}.$$

The first index m can be any integer. If it is not specified, the default is 1:

$$(s_n) = (s_n)_{n=1}^{\infty} = (s_1, s_2, s_3, ..., s_n, ...).$$

The **set of values** of s_n is the set $\{s_1, s_2, s_3, ..., s_n, ...\}$.

Definition A sequence (s_n) **converges to** $s \in \mathbb{R}$ if for every $\epsilon > 0$ there exists an $N \in \mathbb{R}$ such that for all $n > N$, $|s_n - s| < \epsilon$.

We write $s_n \to s$, $\lim_{n\to\infty} s_n = s$, or $\lim s_n = s$. s is called the limit of (s_n).