Lecture 18, 11/4/21
Material corresponds to Ross §33.

Properties of Integrals

Let $f : [a, b] \to \mathbb{R}$ be a function.

Theorem

a) If f is monotone, f is integrable.

b) If f is continuous, f is integrable.

Let $f, g : [a, b] \to \mathbb{R}$ be integrable.

Theorem (linearity)

a) Let $c \in \mathbb{R}$. Then cf is integrable and $\in_a^b cf = f \int_a^b f$.

b) $f + g$ is integrable and $\int_a^b (f + g) = \int_a^b f + \int_a^b g$.

Theorem (order)

a) If $f(x) \leq g(x)$ for all $x \in [a, b]$ then $\int_a^b f \leq \int_a^b g$.

b) $|f|$ is integrable and $\left| \int_a^b f \right| \leq \int_a^b |f|$.

c) If f is continuous, $f(x) \geq 0$ for all $x \in [a, b]$, and $\int_a^b f = 0$ then $f(x) = 0$ for all $x \in [a, b]$.

Theorem Let $f : [a, b] \to \mathbb{R}$ be a function and let $c \in (a, b)$. If $f : [a, c] \to \mathbb{R}$ and $f : [c, b] \to \mathbb{R}$ are integrable then $f : [a, b] \to \mathbb{R}$ is integrable and

$$\int_a^b f = \int_a^c f + \int_c^b f.$$