Lecture 14, 10/19/21
Material corresponds to Ross §22.

Let S be a metric space with distance function d. Let S^* be a metric space with distance function d^*.

Compactness

Theroem A set $E \subset S$ is compact if and only if it is sequentially compact.

Connectedness

Definition A set $E \subset S$ is **disconnected** if there exist open sets $A, B \subset S$ such that

1. $E \subseteq A \cup B$
2. $E \cap A \cap B = \varnothing$
3. $E \cap A \neq \varnothing$
4. $E \cap B \neq \varnothing$.

A set is **connected** if it is not disconnected.

Theorem A set $E \subset \mathbb{R}$ is connected if and only if it is an interval.

Theorem If $E \subset S$ is connected and $f : S \to S^*$ is continuous then $f(E)$ is connected.

Corollary If $E \subset S$ is connected and $f : E \to \mathbb{R}$ is continuous, then $f(I)$ is an interval. In particular, if $a, b \in E$ and $f(a) < y < f(b)$ then there exists $x \in E$ such that $f(x) = y$.