Lecture 12, 10/5/21
Material corresponds to Ross §19.

Uniform Continuity

Definition A function $f : S \to \mathbb{R}$ is **uniformly continuous** if for every $\epsilon > 0$ there exists $\delta > 0$ such that $x, y \in S$ and $|x - y| < \delta$ implies $|f(x) - f(y)| < \epsilon$.

Theorem If $f : [a, b] \to \mathbb{R}$ is continuous, it is uniformly continuous.

Theorem If $f : S \to \mathbb{R}$ is uniformly continuous and (x_n) is a Cauchy sequence in S then $(f(x_n))$ is Cauchy.

Theorem A function $f : (a, b) \to \mathbb{R}$ is uniformly continuous if and only if it can be extended to a continuous function $f : [a, b] \to \mathbb{R}$.