Homework 9

Due Tuesday, November 9 at 10am. Please upload a legible copy to bCourses.

You may work together, but the solutions must be written up in your own words. Show all work and justify all answers.

1. Let $I \subseteq \mathbb{R}$ be an open interval and let $f : I \to \mathbb{R}$ be differentiable such that that $f'(x) \neq 0$ for all $x \in I$.
   a) Prove that there exists an "inverse function", that is, a function $g : f(I) \to \mathbb{R}$ such that $g \circ f(x) = x$ for all $x \in I$ and $f \circ g(y) = y$ for all $y \in f(I)$.

   b) Prove that g is continuous.

2. Ross, 30.2. You may assume that log is continuous (See Problem 1.)

3. a) Let (x_n) be a sequence in $\mathbb{R}$ such that $x_n \to \infty$. Prove that $\frac{1}{x_n} \to 0$.

   b) Let $f : (a, b) \to \mathbb{R}$ be a function such that $f(x) \neq 0$ for all $x \in (a, b)$ and $\lim_{x\to a} f(x) = \infty$. Prove that $\lim_{x\to a} \frac{1}{f(x)} = 0$.

4. Let $a, b \in \mathbb{R}$, $a < b$. Prove that $f : [a, b] \to \mathbb{R}$, $f(x) = x$ is integrable, and find $\int_a^b f$.

5. Ross, 32.6

6. Let $a, b \in \mathbb{R}$, $a < b$. Let $f : [a, b] \to \mathbb{R}$ be a function such that $f(x) = 0$ for all $x \in [a, b]\backslash S$, where S is a finite subset of $[a, b]$. Prove that f is integrable and find $\int_a^b f$.

7. Prove that $f : [0, \pi/2] \to \mathbb{R}$, $f(x) = \sin(x)$ is integrable.