Homework 8

Due Tuesday, November 2 at 10am. Please upload a legible copy to bCourses.

You may work together, but the solutions must be written up in your own words. Show all work and justify all answers.

For problems 1-3, define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2 \sin\left(\frac{1}{x}\right)$ for $x \neq 0$ and $f(0) = 0$.

1. Prove that f is differentiable for $x \neq 0$. Find $f'(x)$ for $x \neq 0$. (Hint: Use chain rule and other theorems, and the fact that $(\sin)' = \cos$).

2. Prove that f is differentiable at 0 and find $f'(0)$. (Hint: use the definition of the derivative.)

3. Prove that $f' : \mathbb{R} \to \mathbb{R}$ is not continuous.

4. a) Use the product rule and induction to show that $(x^n)' = nx^{n-1}$ for all $n \in \mathbb{N}$.

   b) Use the fact that $\left(\frac{1}{x}\right)' = \left(-\frac{1}{x^2}\right)$ and the chain and product rules to prove the quotient rule: If $I \subseteq \mathbb{R}$ is an open interval, $f, g : I \to \mathbb{R}$ are differentiable at $a \in I$, and $g(x) \neq 0$ for $x \in I$, then
   $$\left(\frac{f}{g}\right)'(a) = \frac{f'(a)g(a) - f(a)g'(a)}{(g(a))^2}.$$

5. Ross 29.5

6. Ross 29.13

7. Ross 29.17