Homework 5

Due Tuesday, October 5 at 10am. Please upload a legible copy to bCourses.

You may work together, but the solutions must be written up in your own words. Show all work and justify all answers.

1. Ross 14.1

2. Ross 14.12

3. Ross 14.14 (Note: the given sequence has 2^{n-1} copies of $1/2^n$.)

4. Ross 15.3

5. Use the $\epsilon - \delta$ property to show that the following functions $f : \mathbb{R} \to \mathbb{R}$ are continuous (i.e. for each $x_0 \in \mathbb{R}$, given $\epsilon > 0$, find $\delta > 0$ such that$|x - x_0| < \delta$ implies $|f(x) - f(x_0)| < \epsilon$):
   a) $f(x) = x^2$
   b) $f(x) = x^3$. (Hint: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$)

6. Show that $f : [0, \infty) \to \mathbb{R}$, $f(x) = \sqrt{x}$, is continuous.

7. (Ross 17.10) In each part, prove that $f : \mathbb{R} \to \mathbb{R}$ is is not continuous at $x_0 = 0$.
   a) $f(x) = 1$ for $x > 0$ and $f(x) = 0$ for $x \leq 0$.
   b) $f(x) = \sin(1/x)$ for $x \neq 0$ and $f(0) = 0$.
   c) $f(x) = -1$ for $x < 0$, $f(0) = 0$, and $f(x) = 1$ for $x > 0$.