Homework 10

Due Tuesday, November 16 at 10am. Please upload a legible copy to bCourses.

You may work together, but the solutions must be written up in your own words. Show all work and justify all answers.

- 1. Ross 33.7 and 33.8 (a).
- 2. Define sgn : $\mathbb{R} \to \mathbb{R}$ by sgn(x) = 1 if x > 0, sgn(x) = -1 if x < 0, and sgn(0) = 0. Define $f: [-1,1] \to \mathbb{R}$ by f(x) = x sgn $\left(\sin\left(\frac{1}{x}\right)\right)$ for $x \neq 0$ and f(0) = 0.
 - a) At what points $x \in [-1, 1]$ is f continuous?
 - b) Prove that f is integrable. Hint: f is not piecewise continuous on all of [-1, 1].
- 3. Ross 34.2
- 4. Define $f : \mathbb{R} \to \mathbb{R}$ by f(x) = 0 for x < 0, f(x) = x for $x \in [0, 1]$ and f(x) = 4 for x > 1. Define $F : \mathbb{R} \to \mathbb{R}$ by $F(x) = \int_0^x f$.
 - a) Determine F(x) for each $x \in \mathbb{R}$.
 - b) At which $x \in \mathbb{R}$ is F continuous?
 - c) At which $x \in \mathbb{R}$ is F differentiable? For those x, what is F'(x)?
- 5. Ross 34.5
- 6. Ross 34.7. Indicate precisely how you use change of variables, and check that all conditions of the theorem are met.
- 7. Ross 34.12.