Math H54 | Honors Linear Algebra and Differential Equations | Spring, 2004

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Homework 2 Solutions

1.4, Ex. 10,12,14. Yes (switch rows 1,3); no; no. The matrix in Ex. 12 is a product of two elementary matrices (add 2 times row 2 to row 3, then add 2 times row 1 to row 2).

1.4 Ex. 28: not invertible; **Ex. 32**: the inverse is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -7 & -2 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix}.$$

1.4 Ex. 36. $C^{-1} = \begin{bmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{bmatrix}$. Partitioning into 2×2 blocks and multiplying on each side by C shows that this is correct, and that C is invertible if both A and B are. For the converse, suppose C is invertible. By Theorem (1.50), C is nonsingular, so the only solution to

$$C \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = 0$$

is $\mathbf{x} = \mathbf{y} = 0$. Using block partitioning, we see that

$$C \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} A\mathbf{x} \\ B\mathbf{y} \end{bmatrix},$$

so this shows that both A and B are nonsingular, hence invertible by Theorem (1.50).

1.4 Ex. 37. See book for formula; verify by computing using block partition.

1.4, Ex. 40. $C^5 B^3 A$

1.4, Ex. 49. (a) Using Theorem (1.50), it's enough to show that B is nonsingular. If $\mathbf{x}$ is a non-zero solution of $B\mathbf{x} = 0$, then it's also a solution of $AB\mathbf{x} = 0$. This can't happen, since AB is nonsingular by assumption.

An alternative solution is to use $(AB)^{-1}AB = I$, which shows that $(AB)^{-1}A$ is a left inverse of B. We proved in class that a left inverse of B is a full inverse of B. Let me point out that the theorem from class really is needed for this solution. You can't check directly that $B(AB)^{-1}A = I$, because in order to expand $(AB)^{-1} = B^{-1}A^{-1}$ you have to assume in advance that B and A are invertible, which is what you are trying to prove.

(b) We showed in part (a) that B is invertible. Then B^{-1} is invertible (its inverse is B), hence A is invertible because $A = (AB)B^{-1}$, so it's the product of two invertible matrices.