Homework 12 Solutions

7.4 Ex. 4: Solutions $\mathbf{x}(t)$ of the system correspond to vectors $\begin{bmatrix} y(t) \\ y'(t) \end{bmatrix}$ where $y(t)$ is a solution of the second-order equation. Hence $W[y^{(1)}, y^{(2)}]$ is equal to the Wronskian $W[\mathbf{u}^{(1)}, \mathbf{u}^{(2)}]$ for *some* fundamental set of solutions $\mathbf{u}^{(1)}, \mathbf{u}^{(2)}$ of the system. Now the given fundamental set $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}$ might not be equal to $\mathbf{u}^{(1)}, \mathbf{u}^{(2)}$, but there is a change of basis matrix B such that $[\mathbf{x}^{(1)}, \mathbf{x}^{(2)}] = [\mathbf{u}^{(1)}, \mathbf{u}^{(2)}]\, B$. Then $W[\mathbf{x}^{(1)}, \mathbf{x}^{(2)}] = \det(B) W[y^{(1)}, y^{(2)}]$.

Problem A: $W[\mathbf{y}^{(1)}, \ldots, \mathbf{y}^{(n)}] = \det(B) W[\mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(n)}]$. Thus the two Wronskians differ by a nonzero constant scalar factor.