Math H54 — Honors Linear Algebra and Differential Equations — Spring, 2004

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Homework 11 Solutions

3.2 Ex. 25 doesn't contradict Existence & Uniqueness, because to write it in standard form we must divide by x^2, and then the equation does not have coefficients defined and continuous at $x = 0$.

3.3 Ex. 24. If $y_1(t_0) = y_2(t_0) = 0$, then the Wronskian $W[y_1, y_2]$ evaluated at t_0 is

$$\det \begin{bmatrix} 0 & 0 \\ y_1'(t_0) & y_2'(t_0) \end{bmatrix} = 0.$$

3.5 Ex. 19. Of course the problem really means that a *non-zero* solution can take on the value zero at most once.

If the roots are real and distinct, every solution has the form $y(t) = Ae^{at} + Be^{bt}$ for some $a \neq b$, with A, B not both zero. If $A = 0$, then the solution is never zero. If $A \neq 0$, then $y(t) = 0$ if and only if $Ae^{at} = -Be^{bt}$, if and only if $e^{(a-b)t} = -B/A$, if and only if $t = \ln(-B/A)/(b-a)$. Thus there is exactly one t where $y(t) = 0$ if B/A is negative, and none otherwise.

If the roots are real and equal, every solution has the form $y(t) = e^{at}(At + B)$ with A, B not both zero. Then $y(t) = 0$ if and only if $At + B = 0$, and the equation $At + B = 0$ has exactly one solution if $A \neq 0$, and none if $A = 0$.