Math H54 Honors Linear Algebra and Differential Equations Spring, 2004

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Homework 1 Solutions

1.1, Ex. 30. If $k = -2$ there are infinitely many solutions; otherwise there is exactly one solution.

1.3 Ex. 12. AB is undefined, $BA = \begin{bmatrix} 22 & 12 \\ -10 & -6 \\ 44 & 27 \end{bmatrix}$.

1.3 Ex. 14. Both AB and BA are undefined.

1.3 Ex. 38. The product is $\begin{bmatrix} 11 & -1 & 3 \\ 10 & -6 & -3 \\ 2 & 2 & 3 \end{bmatrix}$.

1.3, Ex. 16. For the given matrices, $(A+B)^2 = \begin{bmatrix} -2 & 9 \\ -6 & -5 \end{bmatrix}$, while $A^2 + 2AB + B^2 = \begin{bmatrix} -4 & 8 \\ -6 & -3 \end{bmatrix}$. The correct general formula is $(A+B)^2 = A^2 + AB + BA + B^2$.

A. Computation shows that $(aI_2 + bJ_2) + (cI_2 + dJ_2) = (a+c)I_2 + (b+d)J_2$, which agrees with the formula $(a+bi) + (c+di) = (a+c) + (b+d)i$ for adding complex numbers. Similarly, using the fact that I_2 is the identity matrix and computing $J_2^2 = -I_2$, we get $(aI_2 + bJ_2)(cI_2 + dJ_2) = acI_2^2 + adI_2J_2 + bcJ_2I_2 + bdJ_2^2 = (ac-bd)I_2 + (ad+bc)J_2$. This agrees with the formula $(a+bi)(c+di) = (ac-bd) + (ad+bc)i$ for multiplying complex numbers.

B. Solve for each column of X separately by Gaussian elimination to get $X = \begin{bmatrix} 2 & -2 \\ 3 & 0 \\ 1 & 2 \end{bmatrix}$.