Math H54 Honors Linear Algebra and Differential Equations Spring, 2004

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Solutions to review problems for Midterm 1

1.

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} -1 & -2 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

2. With x_2 and x_5 as free variables, the solution is

$$\mathbf{x} = \begin{bmatrix} \frac{11}{4} - \frac{x_2}{4} \\ x_2 \\ \frac{2}{3} + \frac{2\,x_5}{3} \\ \frac{5}{3} + \frac{2\,x_5}{3} \\ x_5 \end{bmatrix}.$$

3.

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}; \quad M^{100} = \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 200 & 0 & 1 \end{bmatrix}.$$

4. All except (b).

5. If AB is invertible, then $\det(AB) = \det(A)\det(B) \neq 0$. Hence $\det(A) \neq 0$ and $\det(B) \neq 0$, so A and B are invertible.

6. (a) The matrix A^T is $n \times m$, so the theorem from the lecture implies that A^T has no left inverse. But if $AB = I_m$, then $B^T A^T = I_m$, and we have just argued that this is impossible.

(b) The simplest example is $A = \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right]$. A left inverse is $B = \left[\begin{smallmatrix} 1 & 0 \end{smallmatrix}\right]$, since $BA = [1] = I_1$.

7. Yes, because $(A^{-1})^T = (A^T)^{-1} = A^{-1}$.

8. $\det(A) = \det(B) = \det(C)$ because we can get B from A by row operations (subtract row 1 from each of the other rows) and C from B by column operations (add each of the other columns to column 1). Since C is upper triangular, $\det(C) = 15 \cdot 5^4$.

9. The determinant is $\cos^2\theta + \sin^2\theta = 1$. Using the formula $A^{-1} = (1/\det(A))\operatorname{adj}(A)$, the inverse is

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

10.

$$AB = \begin{bmatrix} 0 & XX^T + I_m \\ X^T X + I_n & 0 \end{bmatrix}$$

11.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1/2 \\ 3 \end{bmatrix}.$$