Math H54   Honors Linear Algebra and Differential Equations   Spring, 2004
Prof. Haiman

Homework 9
due Friday, April 9

Reminder: **Midterm 2** is Tuesday, April 13, in class. Jiangang will hold a review session Sunday, April 11, 3–5 pm in 5 Evans Hall.

Reading for Lectures 19–20:

- Sections 4.4, 4.5

Problems:

- 4.4, Ex. 6, 9, 11, 21

- 4.5, Ex. 4, 12, 16

- Problem A. Show that the QR factorization of a nonsingular square matrix A is unique up to signs. More precisely, if

$$A = Q_1 R_1 = Q_2 R_2$$

  are two such factorizations, show that there is a diagonal matrix D with diagonal entries ± 1, such that
$$Q_2 = Q_1 D, \quad R_2 = D R_1.$$
  (Note that $D^2 = I$, so this will give $Q_2 R_2 = Q_1 R_1$, as it should.)

  Hint: first rearrange the equation $Q_1 R_1 = Q_2 R_2$ to get an identity equating an orthogonal matrix with an upper triangular matrix. Then show that if a matrix is both orthogonal and upper triangular, it must diagonal, with diagonal entries ± 1.

- Problem B. (a) Show that if Q has orthonormal columns, then QQ^T is the matrix of the projection on $CS(Q)$.

  (b) Let Q_1 and Q_2 be two $m \times n$ matrices, each with orthonormal columns. Prove that if $CS(Q_1) = CS(Q_2)$, then $Q_1^T Q_2$ is an orthogonal matrix.

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