Math H54 Honors Linear Algebra and Differential Equations Spring, 2004
Prof. Haiman

Homework 8
due Friday, April 2

Additonal reading for earlier lectures:

- Section 5.5 (for the Fibonacci sequence application)
- Section 4.7 (more about coordinate matrices of linear transformations)

Reading for Lectures 17–18:

- Section 3.2, the part about Dot Product, Norm, and Length
- Section 3.3, Example 10
- Sections 4.2, 4.3

Problems:

- 4.7, Ex. 11. Do part (c) two ways and verify that both give the same answer.
- 3.2, Ex. 23
- 4.2, Ex. 9–11
- 4.2, Ex. 41–42
- 4.3, Ex. 4, 20
- Problem A. (a) Use the formula for the k-th Fibonacci number F_k to derive the identities

$$F_{2k} = 2F_{k+1}F_k - F_k^2$$
$$F_{2k+1} = F_{k+1}^2 + F_k^2.$$

Then compute F_{32}, using only F_1, F_2, F_3, F_4, F_5, F_8, F_9, F_{16} and F_{17} as intermediate results.

(b) Given the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix},$$

show that

$$A^k = \begin{bmatrix} F_{k+1} & F_k \\ F_k & F_{k-1} \end{bmatrix}$$

for all $k > 0$. Use this to derive the identities in part (a) another way.