Math H54 Honors Linear Algebra and Differential Equations Spring, 2004

Prof. Haiman

Homework 4

due Friday, Feb. 20

Reminder: Midterm 1 is Tuesday, Feb. 24, in class. It will cover the material on homework assignments 1 through 4. I'll post some review problems later this week.

Reading for Lectures 8–9:

- Sections 2.2–2.4

Problems:

- 2.3, Ex. 26, 28
- 2.4, Ex. 7, 19, 28, 33
- Problem A. Let A be the matrix in Section 2.4, Ex. 22.

  (a) Reasoning from the signed permutation sum formula, but without writing it out in full, argue that $\det(A)$ must be a polynomial $f(w,x,y,z)$ in the variables w,x,y,z, in which every term has total degree equal to 6.

  (b) Use the properties of determinants to show that $f(w,x,y,z)$ becomes zero whenever any two of the variables w,x,y,z are equal.

  (c) Part (b) implies that each of the polynomials $x-w$, $y-w$, $z-w$, $y-x$, $z-x$ and $z-y$ is a factor of $f(w,x,y,z)$, and hence $f(w,x,y,z)$ is a multiple of the polynomial $d(w,x,y,z) = (x-w)(y-w)(z-w)(y-x)(z-x)(z-y)$. Use part (a) to show that $f(w,x,y,z)$ must be a *constant* multiple $c \cdot d(w,x,y,z)$. Then show that the constant c is equal to 1 [hint: consider the coefficient of xy^2z^3].

- Problem B. Let A_n be the matrix with 1's on the diagonals immediately above and below the main diagonal, and zeroes elsewhere, for example

$$A_5 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

  Show that $\det(A_n)$ is equal to $(-1)^{n/2}$ if n is even, and zero if n is odd.