Math H54 — Honors Linear Algebra and Differential Equations — Spring, 2004

Prof. Haiman

Homework 1

due Friday, Jan. 30

Reading for Lectures 1-3:

- Hill, sections 1.1–1.4

Problems:

- Section 1.1, Ex. 30
- Section 1.2, Ex. 17, 19 (these have answers in the book; show the steps in your solution)
- Section 1.3 Ex. 12, 14, 38
- Section 1.3, Ex. 16. Continue this exercise by finding the correct general formula for $(A+B)^2$ in terms of the matrices A^2, B^2, AB and BA.

A. Let $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ be the 2×2 identity matrix and let $J_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Show that the sum and product of two matrices $aI_2 + bJ_2$ and $cI_2 + dJ_2$ has again the same form, and that the formulas for the sum and product match those for the complex numbers $a + bi$ and $c + di$. (This provides one way of *defining* the complex number system, as an algebra of 2×2 matrices of real numbers.)

B. Let

$$A = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 6 & 0 \\ 2 & -2 \\ 4 & -4 \end{bmatrix}.$$

Solve the matrix equation $AX = B$ for a 3×2 matrix X. Hint: think about the expansion by columns of the matrix product AX.