6. Extending the difference table from the bottom gives

\[
\begin{array}{cccccccc}
1 & 1 & 2 & 3 & 3 & 1 & -4 & -13 & -27 \\
0 & 1 & 1 & 0 & -2 & -5 & -9 & -14 \\
1 & 0 & -1 & -2 & -3 & -4 & -5 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 \\
\end{array}
\]

7. From the left column of the above difference table we get

\[
f(x) = C_0(x) + C_2(x) - C_3(x) = 1 - \frac{5x}{6} + x^2 - \frac{x^3}{6}
\]

13. Make difference tables \((\mod 11)\) for the sequences \(R_i\) and \(iR_i\):

\[
\begin{array}{cccc}
9 & 2 & 9 & 1 \\
4 & 7 & 3 & 6 \\
3 & 7 & 3 \\
4 & 7 \\
3 \\
\end{array}
\quad \begin{array}{cccc}
0 & 2 & 7 & 3 \\
2 & 5 & 7 & 3 \\
3 & 2 & 7 \\
10 & 5 \\
6 \\
\end{array}
\]

From the last rows, the coefficients of \(E(t) = v_0 + v_1 t\) satisfy

\[
\begin{bmatrix}
3 \\
6
\end{bmatrix}
\begin{bmatrix}
v_0 \\
v_1
\end{bmatrix} = 0.
\]

For a solution take \(v_1 = 1\), so \(v_0 = 9\), and \(E(t) = t + 9\). To find \(Q(t)\), make a difference table for \(Q(i) = R_i E(i)\):

\[
\begin{array}{cccc}
4 & 9 & 0 & 1 \\
5 & 2 & 1 & 2 \\
8 & 10 & 1 \\
2 & 2 \\
\end{array}
\]

Then \(Q(t) = 4C_0(t) + 5C_1(t) + 8C_2(t) + 2C_3(t) = 4 + 9t + 3t^2 + 4t^3\), and \(P(t) = Q(t)/E(t) = 9 + 4t^2\). The message was therefore \([9 \ 0 \ 4]\). To check this, encode the message:

\[
\begin{bmatrix}
9 & 0 & 4 \\
1 & 1 & 1 \\
0 & 1 & 4 \\
1 & 1 & 1 \\
\end{bmatrix}
\begin{bmatrix}
1 \\
1 \\
2 \\
3 \\
4
\end{bmatrix} = \begin{bmatrix}
9 \\
2 \\
3 \\
1 \\
7
\end{bmatrix}.
\]

This differs from the received vector by an error in the middle position \((i = 2)\).

To do the above problem using the key equation directly we put \(E(t) = v_0 + v_1 t\) and \(Q(t) = u_0 + u_1 t + u_2 t^2 + u_3 t^3\). Then setting \(Q(i) = R_i E(i)\) for \(i = 0, 1, \ldots, 4\) gives the system
of equations

\[
\begin{bmatrix}
1 & 0 & 0 & 2 & 0 \\
1 & 1 & 1 & 9 & 9 \\
1 & 2 & 4 & 8 & 2 \\
1 & 3 & 9 & 5 & 10 \\
1 & 4 & 5 & 9 & 4
\end{bmatrix}
\begin{bmatrix}
u_0 \\
u_1 \\
u_2 \\
u_3 \\
v_0 \\
v_1
\end{bmatrix} = 0.
\]

Row-reducing (mod 11) gives the equivalent system

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 7 \\
0 & 1 & 0 & 0 & 2 \\
0 & 0 & 1 & 0 & 8 \\
0 & 0 & 0 & 1 & 7 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
u_0 \\
u_1 \\
u_2 \\
u_3 \\
v_0 \\
v_1
\end{bmatrix} = 0,
\]

with a solution \([u_0 \ u_1 \ u_2 \ u_3 \ v_0 \ v_1] = [4 \ 9 \ 3 \ 4 \ 9 \ 1]\), so \(Q(t) = 4 + 9t + 3t^2 + 4t^3\) and \(E(t) = 9 + t\). The rest is the same as in the other solution.

14. To handle an error rate of 1/3 means we need \(e = n/3\). Since \(n = m + 2e\) for a Reed-Solomon code, that implies \(m = n/3\). Hence the data rate for a Reed-Solomon code that can correct \(n/3\) errors in \(1/3\). A triple redundancy code with \(n = 3\) and \(m = 1\) corrects one error in three symbols, so it also has a data rate of 1/3 and tolerates an error rate of 1/3.

The difference is that the Reed-Solomon code can have more than \(n = 3\) symbols in one code block. For example, a Reed-Solomon code of length 150 can correct errors in any 50 of the 150 symbols. A triple-redundancy code with 50 blocks of length 3 carries the same number of message symbols (50), but it can correct 50 errors only if they happen to distribute themselves evenly with one error in each group of 3 symbols, an unlikely possibility.