Math 55, Discrete Mathematics—Spring 2021
Midterm Exam 1

Instructions

The due deadline for this exam is Tuedsday, Feb 23 at 8:00pm PST. This is a firm deadline! Be sure to finish your work with enough time to spare for submitting it.

Points will be deducted for late submissions. Gradescope will not accept any submissions after 11:59pm PST.

The exam is to be completed in a single 80 minute session at any time during the 24-hour window from the release time to the due time. The 80 minute time limit is not enforced, but please do your best to honor it. I have tried to design the exam so that 80 minutes should be enough to demonstrate your knowledge.

If you need to take a few extra minutes to finish answering a question or questions that you know how to solve, that is OK. You should not attempt to use the unenforced time limit as a way to cram in extra studying after seeing the questions. Doing so would be stressful and unlikely to significantly improve your score.

You can write answers on paper and scan in PDF format (best) or take pictures (usually lower image quality, so be sure they are readable). You can also write answers on a tablet if you have one, and save them in PDF format.

When you submit your answers, Gradescope will ask you to indicate the page or pages where your answer to each question is located. Gradescope will let you indicate one or multiple questions on each page.

This exam is open book. You may consult the textbook, your own notes, and any other books and non-interactive web resources. You may not receive assistance from another person, or give assistance, or use web resources that provide answers to questions interactively.

There are 8 questions, for a total of 100 points. Questions are on the next page.

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Question 1 (3 points for each correct match). Match each symbolic proposition (a)–(e) with the number of the phrase that applies to it.

(a) $q \to p$	1) is a tautology
(b) $q \vee \neg p$	2) is the contrapositive of (a)
(c) $(p \wedge q) \vee (\neg p \wedge \neg q)$	3) is equivalent to p implies q
(d) $\neg p \to \neg q$	4) is equivalent to p if and only if q
(e) $(p \wedge q) \to (p \vee q)$	5) is the symbolic expression for "q only if p"

Question 2 (13 points). Prove that if B and C are subsets of a set A, then

$$C \cap (A - B) = C - B.$$

Question 3 (4 points per part). What property of a function $f : X \to Y$ is expressed by each of the following propositions?

(a) $\forall y \in Y \, \exists x \in X \, (f(x) = y)$

(b) $\forall y \in Y \, \exists! x \in X \, (f(x) = y)$

(c) $\exists x \in X \, \exists z \in X \, (x \neq z \wedge f(x) = f(z))$

Question 4 (12 points). Is the product of two irrational numbers always irrational? Prove or give a counterexample.

Question 5 (12 points). Show that the set of irrational real numbers is uncountable. You can take as known all properties of countable and uncountable sets shown in class or the book.

Question 6 (12 points). Use the Euclidean algorithm to find $\gcd(105, 826)$. Show the steps.

Question 7 (12 points). Find the least common multiple of $2^{15} \cdot 7^3 \cdot 11^2$ and $2^6 \cdot 5 \cdot 7^4$. Express your answer as a product of powers of primes, in the same manner as the two given numbers.

Question 8 (12 points). What set A has power set $P(A) = \{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}, \{\varnothing, \{\varnothing\}\}\}$?