

Question 1. [15 pts] A shop has eight flavors of bagels. Only 4 raisin bagels and 6 onion bagels are available. Each of the other six flavors is available in an unlimited amount. How many different assortments of 20 bagels can you order?

$$\binom{27}{20} - \binom{22}{15} - \binom{20}{13} + \binom{15}{8}$$

First term is choices with no restrictions.

Subtract terms for ≥ 5 raisin ~~and~~ or ≥ 7 onion.

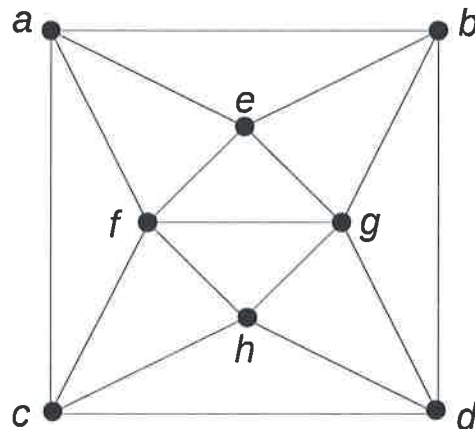
Add back term for ≥ 5 raisin and ≥ 7 onion.

Question 2. [15 pts] January 1, 1999 was a Friday. Find the probability that a randomly chosen person born in 1999 was born on a Monday. Assume that all 365 days in 1999 are equally likely as birthdays.

$365 = 52 \times 7 + 1$, so each day of week occurs 52 times in the year, except for one which occurs 53 times. The excess day is the first (and last) day; in this case Friday.

$$\text{So } p(\text{born on Monday}) = \frac{52}{365}.$$

Question 3.



(a) [10 pts] If the graph shown above has an Eulerian **circuit**, find one, and describe it by listing all vertices along the circuit. If the graph does not have an Eulerian circuit, prove that it does not.

No Eulerian circuit, because not every vertex has even degree.

(b) [10 pts] If the graph shown above has an Eulerian **path**, find one, and describe it by listing all vertices along the path. If the graph does not have an Eulerian path, prove that it does not.

One such path is

f e a b d c a f c h d g b e g h f g .

There are many others. All of them have endpoints f and g, as these are the only two odd degree vertices.

Question 4. (a) [10 pts] Find an example of a simple undirected graph for which the list of vertex degrees is $(3, 3, 3, 3, 1, 1)$. Express your answer by drawing a picture of your graph.



(b) [15 pts] Find, and draw, a different simple undirected graph with vertex degrees $(3, 3, 3, 3, 1, 1)$, which is not isomorphic to your graph in part (a). Give a reason showing that the two graphs are not isomorphic.



Not isomorphic because one is connected and the other is not.

Swapping the two graphs also gives a correct answer.

Question 5. Let E and F be events in a sample space S .

(a) [12 pts] Show that $p(\bar{E} \cap F) + p(E \cap F) = p(F)$.

$\bar{E} \cap F$ and $E \cap F$ are disjoint (since one is contained in \bar{E} and the other in E) and their union is F (since every $x \in F$ is either in E , hence in $E \cap F$, or in \bar{E} , hence in $\bar{E} \cap F$).

Therefore $p(\bar{E} \cap F) + p(E \cap F) = p(F)$.

(b) [13 pts] Prove that if E and F are independent events, then \bar{E} and F are independent.

$$\begin{aligned} p(\bar{E} \cap F) &= p(F) - p(E \cap F) && \text{by part (a)} \\ &= p(F) - p(E)p(F) && \text{since } E \text{ and } F \text{ are independent} \\ &= p(F)(1 - p(E)) \\ &= p(F)p(\bar{E}). \end{aligned}$$

This shows that \bar{E} and F are independent.