Math 55—Fall 2012
Homework 14 Solutions

28. Let X_i be 1 if the i-th roll is a 6, or 0 otherwise. It has variance $E(X_i^2) - E(X_i)^2 = 1/6 - 1/36 = 5/36$. The number of 6's in ten rolls is $X = X_1 + \cdots + X_{10}$. Since they are pairwise independent, we can add their variances to get $V(X) = 50/36$.

For the extra part of the problem, note that $EX = 10EX_i = 10/6$. If more than 3 6's appear, that is, if at least 4 6's appear, then $|X - EX| \geq 4 - 10/6 = 14/6$. By Chebyshev's inequality, $P(|X - EX| \geq 14/6) \leq V(X)/(14/6)^2 = 50/196 = 25/98$, or about .255. Note that this is an upper bound on the probability. Using the binomial distribution we can calculate its exact value to be

$$P(X \geq 4) = \sum_{k=4}^{10} \binom{10}{k} (1/6)^k (5/6)^{10-k} = 29279/419904 \approx .0697$$

38(b). By Chebyshev, $P(|X - 10000| \leq 1000) = 1 - P(|X - 10000| \geq 1000) \geq 1 - 1000/(1000^2) = .999$.