Math 55—Fall 2012
Homework 13 Solutions

7.4 #4. $10 \cdot (.6) = 6$.

7.4 # 16. To prove X and Y are not independent it is enough to prove, for instance, that the events $X = 0$ and $Y = 0$ are not independent. Since $X + Y = 2$, the event $(X = 0) \wedge (Y = 0)$ is impossible, thus $P((X = 0) \wedge (Y = 0)) = 0$. But $P(X = 0) = 1/4$ and $P(Y = 0) = 1/4$, so $P((X = 0) \wedge (Y = 0)) \neq P(X = 0)P(Y = 0)$.

7.4 #26. If $X(s) = k$, the sample point s belongs to the events A_1 through A_k but not to A_j for $j > k$. In particular, s belongs to exactly k of the events A_j. In the sum $\sum_{k=1}^{\infty} p(A_k)$, if we replace $p(A_k)$ by its definition as the sum of $p(s)$ over all $s \in A_k$, we see that each term $p(s)$ occurs $X(s)$ times. Hence the sum is equal to $\sum_{s \in S} p(s)X(s)$, which is EX.

7.4 #38(a). Let X be the random variable which is the number of cans filled in the day. We are given $EX = 10,000$, and we clearly have $X \geq 0$. By Markov's inequality, it follows that $P(X \geq 11,000) \leq 10,000/11,000 = 10/11$.

Chapter 7 Supp. Ex. 22 (a) There are b bins, and the ball is equally likely to fall into any of them, so the probability of it landing in a specified bin is $1/b$.

(b) If Y_i is the indicator random variable for the event that the i-th ball lands in the specified bin, then $EX_i = 1/b$, by part (a). The number of balls that land in this bin is $Y = Y_1 + \cdots + Y_n$. By linearity of expectation, $EY = n/b$.

(c) The number of balls tossed until one lands in a specified bin is geometricallly distributed with parameter $p = 1/b$. Hence its expectation is b.

(d) Once $i - 1$ bins contain balls, the probability on each toss of landing in a new bin is $(b - i + 1)/b$. The number of tosses X_i to occupy an i-th bin once $i - 1$ have been occupied is therefore geometrically distributed, with expectation $EX_i = b/(b - i + 1)$. The number of tosses to occupy all the bins is $X = X_1 + \cdots + X_b$, with

$$EX = \sum_{i=1}^{b} \frac{b}{b - i + 1} = b\left(1 + \frac{1}{2} + \cdots + \frac{1}{b}\right).$$