Math 55—Fall 2012
Homework 12 Solutions

7.2 #18. (a) 1/7, because after choosing the first person, the second person has a one in seven chance to have been born on the same day of the week.

(b) For $n > 7$, two must be born on the same day of the week by the pigeonhole principle, so the probability is 1. For $2 \leq n \leq 7$, there are 7^n possible functions assigning each person a day of the week to have been born on, all equally likely. Of these, there are $P(7, n) = 7 \cdot 6 \cdot \cdots \cdot (7 - n + 1)$ ways for all n people do have been born on different days of the week. Therefore, the probability that two were born on the same day is $1 - P(7, n)/7^n$.

(c) Calculating $p(n) = 1 - P(7, n)/7^n$ for $2 \leq n \leq 7$, we get

n	2	3	4	5	6	7
$p(n)$	$\frac{1}{7}$	$\frac{19}{49}$	$\frac{223}{343}$	$\frac{2041}{2401}$	$\frac{16087}{16807}$	$\frac{116929}{117649}$

This is greater than 1/2 for $n \geq 4$.

7.2 #30. (a) $(1/2)^{10}$

(b) $(.6)^{10}$

7.3 #10. (a) $(.97)(.04)/((.97)(.04) + (.02)(.96))$, or approximately .67.

(b) $1 - p$, where p is the answer to part (a), approximately .33.

(c) $(.03)(.04)/((.03)(.04) + (.98)(.96))$, or approximately .0013.

(d) $1 - q$, where q is the answer to part (c), approximately .9987.

7.3 #14. $p(F_2|E) = p(E|F_2)p(F_2)/\sum_i p(E|F_i)p(F_i) = (3/8)(1/2)/((2/7)(1/6) + (3/8)(1/2) + (1/2)(1/3)) = 7/15$.

Additional problem: $C(10, 4)(1/6)^4(5/6)^6$.