

7.1 #10. $C(51, 3) / C(52, 5)$

#28. $C(11, 7) / C(80, 7)$

#36. With two dice, there are 5 ways to roll a total of 8:
 $2+6, 3+5, 4+4, 5+3, 6+2$, for a probability of
 $5/36 \approx .139$

With three dice, there are 21 ways to roll 8:

$1+1+6, 1+2+5, \dots, 1+6+1 \leftarrow (6)$

$2+1+5, 2+2+4, \dots, 2+5+1 \leftarrow (5)$

$3+1+4, 3+2+3, 3+3+2, 3+4+1 \leftarrow (4)$

$4+1+3, 4+2+2, 4+3+2 \leftarrow (3)$

$5+1+2, 5+2+1 \leftarrow (2)$

$6+1+1 \leftarrow (1)$

for a probability of $21/6^3 = 21/216 \approx .097$

So it is more probable to roll 8 with two dice than with three.

#38. (a) Independent: $p(E_1) = p(E_2) = \frac{1}{2}$, $p(E_1 \cap E_2) = \frac{1}{4}$

(b) ~~Independent~~. $p(E_1) = \frac{1}{2}$, $p(E_2) = \frac{2}{8} = \frac{1}{4}$,
 $p(E_1 \cap E_2) = \frac{1}{8}$

(c) Not independent. $p(E_1) = \frac{1}{2}$, $p(E_2) = \frac{1}{4}$,
 $p(E_1 \cap E_2) = 0$.

7.2 #8. a) $\frac{1}{2}$ b) $\frac{1}{2}$

c) Think of 12 as a single symbol: there are $(n-1)!$ permutations ~~size~~ in which 1 immediately precedes 2, hence the probability is $P = \frac{(n-1)!}{n!} = \frac{1}{n}$

d) This means the four numbers 1, 2, $n-1$, n are in one of the orders

$$n 1 n-1 2, \quad n n-1 1 2, \quad n n-1 2 1,$$

$$n-1 n 1 2, \quad n-1 2 n 1, \quad n-1 2 n 1.$$

Since these four numbers are equally likely to appear in any of 24 orders, $P = 6/24 = \frac{1}{4}$.

e) This means 1, 2 and n are in one of the orders

$$n 1 2, \quad n 2 1$$

$$P = 2/6 = \frac{1}{3}$$

#16. We are given that $P(E \cap F) = P(E)P(F)$. Then

$$\begin{aligned} P(\bar{E} \cap \bar{F}) &= P(\overline{E \cup F}) = 1 - P(E \cup F) = \\ &= 1 - (P(E) + P(F) - P(E \cap F)) = 1 - P(E) - P(F) + P(E)P(F) \\ &= (1 - P(E))(1 - P(F)) = P(\bar{E})P(\bar{F}). \end{aligned}$$

#22 a) The rules for leap years imply that 97 out of every 400 years are leap years. Thus the probability we assign to being born on Feb. 29 should be $\frac{97}{400}$ times the probability we assign to ~~any~~ each of the remaining 365 days. If we call p the probability assigned to the other days, and q the probability assigned to Feb. 29, this means

$$q = \frac{97}{400} p$$

$$365p + q = 1$$

Solving, we get $p = \frac{400}{146,097}$, $q = \frac{97}{146,097}$.

$$\#28. \quad a) \quad C(5,3) \cdot (.51)^3 \cdot (.49)^2$$

$$b) \quad 1 - (.49)^5$$

$$c) \quad 1 - (.51)^5$$

$$d) \quad (.51)^5 + (.49)^5$$

#38. a) We want the conditional probability

$P(E|F)$ where $E = \{\text{total is seven}\}$
 $F = \{\text{at least one six}\}.$

Then $P(E \cap F) = \frac{2}{36} \quad (1+6 \text{ or } 6+1)$

$$P(F) = \frac{11}{36} \quad (6+x \text{ or } 6x+6 \text{ makes } 12, \text{ but } 6+6 \text{ is included in both})$$

$$P(E|F) = P(E \cap F)/P(F) = 2/11$$

b) Replace F by $F' = \{\text{at least one five}\}.$

$$\text{Again } P(F') = 11/36, \text{ and } P(E \cap F') = \frac{2}{36} \quad (2+5 \text{ or } 5+2).$$

$$\text{So } P(E|F') = 2/11, \text{ same as } P(E|F).$$