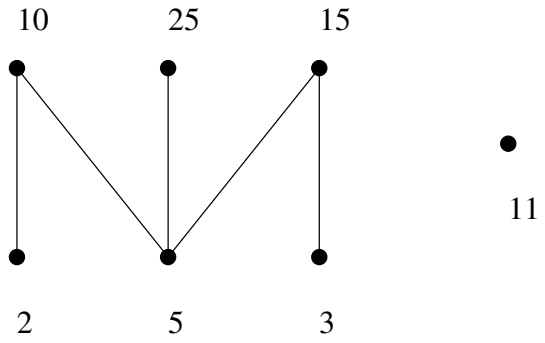
Math 55: Discrete Mathematics, Fall 2008
Homework 14 Solutions

8.6: 6. (a) and (c) are posets, (b) and (d) are not.

22 (c)

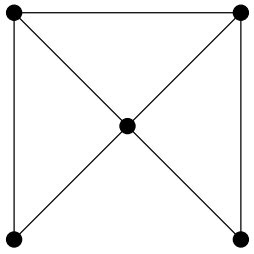


36 (a) $(\mathbb{R}_{\geq 0}, \leq)$
(b) $(\mathbb{R}_{\geq 0}, \geq)$
(c) $(\mathbb{R}, \leq)$.

9.2: 18. Suppose there are n vertices. The possible degrees are then 0 through $n-1$. If no two vertices have the same degree, there must be one of each degree. But if there is a vertex of degree $n-1$, then every other vertex is adjacent to it, so there is no vertex of degree 0, if $n \geq 2$.

26(a) K_n is bipartite for $n = 1$ or 2.
(b) C_n is bipartite for n even.

34. It has $(4+3+3+2+2)/2 = 7$ edges.



* 58. Suppose the two parts have k and l vertices respectively. Then $k + l = v$, and $e \leq kl$. Therefore $v^2/4 - e \geq (k+l)^2/4 - kl = (k-l)^2/4 \geq 0$, so $e \leq v^2/4$.

9.3: 38. Isomorphic, via $u_1 \mapsto v_1$, $u_2 \mapsto v_3$, $u_3 \mapsto v_2$, $u_4 \mapsto v_5$, $u_5 \mapsto v_4$ (other isomorphisms also exist).

44. Not isomorphic. One way to distinguish them is that in the v graph, the five vertices adjacent to each vertex v_i contain a circuit of length 4, while in the u graph, they do not.

* 9.4: 18. G has a Hamiltonian circuit (i.e., one passing through all the vertices) but H does not.

54. There are 10 allowable states. The graph is the union of two paths of length 8 from the initial to the finnal state, which overlap in paths of length 3 at the beginning and the end. Each path is thus a solution in seven crossings. Part (e) of the problem seems to be a mistake, as both solutions have the same number of crossings with an animal.

9.7: 6. A planar drawing can be made by switching the positions of c and f, drawing af above b, and cd below e.

* 8. Not planar, because the induced subgraph on vertices a through f is a $K_{3,3}$.

9.8: 12 (for graphs 5 and 9 only). Graph 5 has chromatic number 3, which decreases to 2 if you remove a, b or c. Graph 9 has chromatic number 2, and it clearly cannot be decrease to 1 by removing a vertex.

18. The graph has chromatic number 3, hence 3 channels are needed.

(A) The linear extensions are $\{a, b, c\}$ in any order, followed by g, followed by $\{d, e, f\}$ in any order. Thus there are $6 \times 6 = 36$ of them.

(B) One example is the wheel W_5. Since the outer circuit has odd length, it needs 3 colors, and then the central vertex needs a fourth color.