

Math 55: Discrete Mathematics, Fall 2008
Reading and Homework Assignment 13

Reading:

Lecture 36: 8.5

Lecture 37: 5.5, Examples 10-11; 7.6, Theorem 1 and Examples 2-3

Homework (due Monday, 12/1):

Self-checking problems

8.5: 3, 15, 21-22, 39, 45, 51

Problems to hand in:

8.5: 16, 24(a,b), 40, 46, 54

- (A) Prove that the smallest equivalence relation containing a given relation R is the transitive closure S^* of the reflexive and symmetric closure $S = R \cup R^{-1} \cup \Delta$.
- (B) Find an example of a relation R on a set with 3 elements such that the reflexive and symmetric closure of the transitive closure R^* is not an equivalence relation.
- (C) Give a combinatorial proof that the Stirling numbers $S(n, k)$ satisfy the following analog of Pascal's relations:

$$S(n, k) = S(n - 1, k - 1) + kS(n - 1, k)$$

for $k, n > 0$. [Hint: if X denotes the set of partitions of $[n]$ into k parts, consider the decomposition $X = A \cup B$, where A consists of partitions in which $\{n\}$ is a block, and B consists of partitions in which n belongs to a block containing at least one other element.] Use the above relation and the initial conditions $S(n, 0) = 1$ if $n = 0$, $S(n, 0) = 0$ if $n > 0$ to make a table of $S(n, k)$ for n and k less than or equal to 7. Check by comparing the value for $S(7, 3)$ in your table to the value given by the formula in Chapter 5.5.