Math 55: Discrete Mathematics, Fall 2008
Homework 12 Solutions

7.5: 8. $270 - 64 - 94 - 58 + 26 + 28 + 22 - 14 = 116$

* 14. $26! - 23! - 23! - 24! + 21!$. In this problem our three sets are $A_{\text{fish}} = \{\text{strings containing } \textit{fish}\}$, and A_{rat}, A_{bird} defined similarly. We get $|A_{\text{fish}}| = 23!$ by treating *fish* as a single letter, and the others likewise. We get $|A_{\text{fish}} \cap A_{\text{rat}}| = 21!$ by treating both *fish* and *rat* as single letters. The other intersections are empty, because *bird* has letters in common with both *fish* and *rat*.

20. $5 \cdot 10^4 - \binom{5}{2}10^3 + \binom{5}{3}10^2 - \binom{5}{4}10 + \binom{5}{5}1$. By the binomial theorem, this is equal to $(1-10)^5 + 10^5 = 10^5 - 9^5 = 40951$.

7.6: 6. The non-square-free positive integers < 100 are divisible by 2^2, 3^2, 5^2 or 7^2. Let A_p be the set of those divisible by p^2. Note that $A_p \cap A_q = A_{pq}$ since p and q are relatively prime, and likewise for products of more than two factors. Moreover, if the product is ≥ 10, then the intersection is empty, so the only non-empty intersection that contributes to the answer is $A_2 \cap A_3 = A_6$.

Therefore our answer is $99 - |A_2| - |A_3| - |A_5| - |A_7| + |A_6| = 99 - 24 - 11 - 3 - 2 + 2 = 61$.

* 8. $5^7 - \binom{5}{1}4^7 + \binom{5}{2}3^7 - \binom{5}{3}2^7 + \binom{5}{4}1^7$

14. $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!} + \frac{1}{8!} - \frac{1}{9!} + \frac{1}{10!}$

* [2.5 pts each part] 8.1: 4(a) transitive, antisymmetric (b,c) reflexive, symmetric and transitive, (d) reflexive, symmetric

50. The relation $R \cap R^{-1}$ is the set of pairs (a, b) such that both (a, b) and (b, a) belong to R. The definition of the antisymmetric property is that every such pair has $a = b$, which is equivalent to the assertion that (a, b) is in the diagonal relation Δ. Hence R is antisymmetric iff $R \cap R^{-1} \subseteq \Delta$.

8.3: 32. The definitions of the terms *irreflexive* and *asymmetric* aren't easy to find in the book (they're in 8.1, Exercises 9-22), so we'll only require that you specify the other properties.

The digraph in 26 is reflexive and not symmetric, antisymmetric or transitive.

In 27, symmetric, but not reflexive, antisymmetric or transitive.

In 28, reflexive, symmetric, transitive, not antisymmetric.

8.4: 22. Since R^* contains R, if R is reflexive then so is R^*.

28 (for 26(c)). The transitive closure is the total relation $X \times X$ on $X = \{a, b, c, d, e\}$.