**Math 55: Discrete Mathematics, Fall 2008**
**Homework 10 Solutions**

3.8: 2(b)

$$\begin{bmatrix} -4 & 9 & 2 & 10 \\ -4 & -5 & 4 & 0 \end{bmatrix}$$

Or, in arithmetic mod 7,

$$\begin{bmatrix} 3 & 2 & 2 & 3 \\ 3 & 2 & 4 & 0 \end{bmatrix}$$

* 4(c)

$$\begin{bmatrix} 2 & 0 & -3 & -4 & -1 \\ 24 & -7 & 20 & 29 & 2 \\ -10 & 4 & -17 & -24 & -3 \end{bmatrix}$$

Or, in arithmetic mod 7,

$$\begin{bmatrix} 2 & 0 & 4 & 3 & 6 \\ 3 & 0 & 6 & 1 & 2 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix}$$

* [5 pts each part] (A) (a) $1 - (.95)^4 \approx .185$
(b) $1 - (.95)^7 - 7(.95)^6(.05) \approx .044$

(B) For $p = .05$, $1 - h \approx .713$ is the theoretical maximum data rate. The data rate when 4 message bits are encoded with 7 code bits is $4/7 \approx .571$, quite a bit less than the theoretical maximum.

* (C) If the code can always correct 2 errors, then every word within Hamming distance 2 of a codeword $C$ must decode to $C$. For a 15-bit code, the number of such words is

$$1 + \binom{15}{1} + \binom{15}{2} = 121.$$

Hence the number of codewords is at most $\lfloor 2^{15}/121 \rfloor = 270$. To encode $m$ message bits requires $2^m$ codewords, so $2^m$ can be at most $2^8 = 256$.