Math 55: Discrete Mathematics, Fall 2008
Homework 9 Solutions

* 6.3: 4. Let F be the event that Ann selects an orange ball, E the event that she chooses box 2. Then $p(E) = 1/2$, $p(F|E) = 5/11$, $p(F|\overline{E}) = 3/7$. Bayes' Theorem gives $p(E|F) = (5/11)(1/2)/((5/11)(1/2) + (3/7)(1/2)) = 35/68$.

10. (a) $(.97)(.04)/((.97)(.04) + (.02)(.96)) \sim 0.669$
(b) $(.02)(.96)/((.97)(.04) + (.02)(.96)) \sim .331$
(c) $(.03)(.04)/((.03)(.04) + (.98)(.96)) \sim .00127$
(d) $(.98)(.96)/((.03)(.04) + (.98)(.96)) \sim .9987$

12. (a) $(.9)(2/3) + (.2)(1/3) = 2/3$
(b) $p(\text{send } 0|\text{receive } 0) = p(\text{receive } 0|\text{send } 0)p(\text{send } 0)/p(\text{receive } 0) = .9(2/3)/(2/3) = .9$

14. $(3/8)(1/2)/((2/7)(1/6) + (3/8)(1/2) + (1/2)(1/3)) = 7/15$

6.4: 4. $10(.6) = 6$

8. $3(7/2) = 21/2$

16. $p(X = 0) = 1/4$, $p(Y = 0) = 1/4$, but $p(X = 0|Y = 0) = 0 \neq (1/4)(1/4)$. You could also use other values of these random variables to give a counterexample to independence.

* 30. Let X be the number of tails on n flips. Let X_i be the indicator variable for tails on the i-th flip. Then $E(X_i) = .4$, hence $E(X) = (.4)n$. The flips are independent, and $V(X_i) = .4 - (.4)^2 = .24$, so $V(X) = (.24)n$. By Chebyshev, $p(|X - (.4)n| \geq \sqrt{n}) \leq (.24)n/(\sqrt{n})^2 = .24$.

* 42. m/n. There are two ways to solve this problem using linearity of expectation: (i) The number of balls in bin 1 is the sum of m indicator variables, each with probability and expectation $1/n$, or (ii) by symmetry, every bin has same the expected number of balls, and this expectation multiplied by the number of bins, n, must equal the number of balls, m.

Ch. 6 Suppl. Ex. 16. $\binom{n}{n/2}(1/2)^n$, provided n is even. The probability is zero if n is odd.

18. $1/2^5 = 1/32$. The events that the i-th bit equals the $(12 - i)$-th bit, for $i = 1, \ldots, 5$, are independent and each have probability $1/2$.

24. Given that $p(A)$ and $p(B)$ are non-zero, the inequalities $p(B|A) < p(B)$ and $p(A|B) < p(A)$ are both equivalent to $p(A \cap B) < p(A)p(B)$.

(A) The expected value of the number X of correctly returned hats is 1. Since $X \geq 0$ always, it follows by Markov that $P(X \geq k) \leq 1/k$.